

U S A Mathematical Talent Search

PROBLEMS

Round 1 - Year 11 - Academic Year 1999-2000

- 1/1/11.** The digits of the three-digit integers a , b , and c are the nine nonzero digits 1, 2, 3, ..., 9, each of them appearing exactly once. Given that the ratio $a:b:c$ is 1:3:5, determine a , b , and c .
- 2/1/11.** Let $N = 111...1222...2$, where there are 1999 digits of 1 followed by 1999 digits of 2. Express N as the product of four integers, each of them greater than 1.
- 3/1/11.** Triangle ABC has angle A measuring 30° , angle B measuring 60° , and angle C measuring 90° . Show four different ways to divide triangle ABC into four triangles, each similar to triangle ABC , but with one quarter of the area. Prove that the angles and sizes of the smaller triangles are correct.
- 4/1/11.** There are 8436 steel balls, each with radius 1 centimeter, stacked in a tetrahedral pile, with one ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. Determine the height of the pile in centimeters.
- 5/1/11.** In a convex pentagon $ABCDE$ the sides have lengths 1, 2, 3, 4, and 5, though not necessarily in that order. Let F , G , H , and I be the midpoints of the sides AB , BC , CD , and DE , respectively. Let X be the midpoint of segment FH , and Y be the midpoint of segment GI . The length of segment XY is an integer. Find all possible values for the length of side AE .

Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet and a completed Entry Form, should be sent to the following address and **postmarked no later than September 13, 1999**. Each participant is expected to develop solutions without help from others.

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